Montana Standards for
Mathematics

A Vision for Montana Mathematics

The world as we know it is changing at an ever increasing pace. The teaching of mathematics in Montana public schools needs to be flexible enough to deliver rigorous material that continues to be relevant to the changing lives of our students. In that vein, Montana teachers are challenged to envision the world not as we know it today, but to keep in the forefront of their minds the world our students will be living in tomorrow.

Envision a classroom where instruction is focused on the *big* ideas of mathematics. On a daily basis, students are expected to engage, interact, collaborate, explain, and excel. Envision the powerful students that such an atmosphere will create—students who are active, excited, curious, and confident; students who *learn*. In this classroom, mathematics is more than just content to be studied; it is an activity to be enjoyed.

There are many aspects of our students' school experience that are outside of our control. However, we do have influence over the mathematics we teach and how we teach it. Montana's mathematics teachers are first class. They are innovators. The standards set forth in this document are of the same quality. To bring them to life requires Montana educators to do what they do best; innovate, challenge, and achieve.

Implementing the Vision

The Montana content standards for mathematics are not about mandating curriculum or recommending specific courses in Montana's schools. Instead, they are about preparing students to work and live successfully in a society that is increasingly technical, global, and multicultural. We have set high expectations for the performance of Montana students at all levels; it is the responsibility of local communities and districts to determine the path for their students to achieve the goals set out in this document.

Mathematics Content Standard 1
Number Sense and Operation: A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates and compute fluently within a variety of relevant cultural contexts.

Rationale
Number sense and computational fluency are the foundation for school mathematics and life in a quantitative society that includes multiple cultures. Students who have a sense of quantity are fluent with basic facts; perform mental computations; determine the reasonableness of a solution; and understand that properties of operations have the power to solve problems, and use number to describe the world. The foundation of number sense and operations supports the other content standards.

Benchmarks

	End of 4th grade	End of 8th grade	Upon Graduation
1.1	Place Value: Demonstrate the relationship among whole numbers; identify place value up to 100,000, and compare numbers using greater than, less than, and equal.	Number Relationships: Recognize, model, and compare number relationships including percents and rational numbers, integers, fractions, decimals, and numbers in exponential form.	Quantification: With and without technology perform operations on very large and very small numbers, including small numbers with large magnitude, using multiple notations and interpret their effects in problem situations including those of culture.
1.2	Estimation and Operations: Estimate sums, differences, and products when solving problems. Add, subtract, multiply up to three-digit by two-digit factors, and divide two-digit dividends by one-digit divisors that result in whole number quotients to solve problems.	Estimation: Select and apply appropriate estimation strategies to judge the reasonableness of solutions to problems including those computed on a calculator.	Estimation: Identify situations where estimation is appropriate and determine the needed degree of accuracy for a given problem situation (and the appropriate precision in which to report answers).

1.3	Whole Number Concepts: Develop multiplication and division concepts, reason and justify using number and operation models and strategies, and demonstrate fluency with basic facts and using properties of operations.	Number Theory: Demonstrate the use of number theory concepts such as order of operation and prime factorization in problem situations.	Equivalence: Given a representation of a number or expression, find equivalent representations using multiple notations (e.g., x to the one half is equivalent to the square root of x, or graphical representation of multiplying binomials).
1.4	Fractions/Decimals: Identify and model common fractions (such as, tenths, quarters, thirds, halves) and decimals (such as, money and place value to 0.001) and recognize and compare equivalent representations.	Rational Number Operations: Compute fluently and solve multi-step problems using integers, fractions, decimals, and numbers in exponential form.	Properties: Analyze and apply the properties of numbers and number systems.
1.5	Measurement: Select and apply appropriate standard units and tools to measure weight, time, and temperature including scientific and cultural situations when relevant.	Measurement: Measure using metric and standard units of measurement in contextually relevant problems including cultural situations; compare and convert within systems, and use appropriate technology.	Modeling: Identify givens and unknowns in an unfamiliar situation including those of culture and describe relationships between variables (e.g., the effect of changing an interest rate on monthly payments).
1.6		Proportional Reasoning: Understand and apply proportional relationships to model real world situations to solve problems involving rates, ratios, proportions, percents, and direct variation.	

Mathematics Content Standard 2
Data Analysis: A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts.

Rationale
Data analysis or statistical literacy pertains to all aspects of daily life within multiple cultures. As consumers of information, students who analyze data to make decisions and predictions are better prepared to be responsible citizens. Students who understand and apply basic concepts of probability and make connections to data analysis build strong quantitative reasoning for productive personal and professional lives.

Benchmarks

	End of 4th grade	End of 8th grade	Upon Graduation
2.1	Represent Data: Collect, represent, and organize data in tables, dot plots, bar graphs, pictographs, and stem and leaf plots with and without technology.	Represent Data: Collect, organize, and represent data in box plots, histograms, scatter plots, and circle graphs to include culturally relevant contexts with and without technology.	Represent Data: Select and create graphical or numerical representations for data sets and compare different data sets using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation) with and without technology.
2.2	Evaluate Data: Solve problems and make decisions using data descriptors such as minimum, maximum, median and mode within scientific and cultural contexts when relevant.	Evaluate Data: Compute, interpret, analyze, and evaluate data using mean, median, range, and quartiles to identify trends and make decisions and predictions about data as it relates to multiple contexts.	Evaluate Validity: Evaluate the validity of reports including those culturally relevant based on data collected and/or published by considering the source of the data, the design of the study, and the way data are analyzed and displayed (e.g., correlation does not prove causation).
2.3	Probability: Describe events as likely or unlikely and discuss the degree of	Probability: Create sample spaces, determine experimental and	Probability: Make, evaluate, and justify decisions based on

	likelihood using words such as certain, equally likely, and impossible including cultural context when relevant.	theoretical probabilities (e.g., using tree diagrams), and make predictions using real-life contexts or simulations including those of culture.	probabilities in problem situations including those of culture. (e.g., understanding expected value as in a lottery situation, rules of probability)
2.4			Counting: Determine the possible number of outcomes for an event or compound event using the fundamental counting principle, permutations, combinations and other systematic counting methods. With and without technology analyze the results in a problem situation.)
2.5			Curve Fitting: Find an appropriate model with and without technology for two variable data using curve fitting. Create an equation corresponding to this model and decide when predictions based on this equation are valid.

Mathematics Content Standard 3
Geometric Reasoning: A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts.

Rationale
Geometric reasoning complements the study of number, operations, and probability models. Students who have a sense of space analyze two- and three-dimensional shapes and their properties and relationships, and can make connections within mathematics. Geometric reasoning can help students appreciate and value mathematics and make connections to their world through multiple cultural contexts.

Benchmarks

	End of 4th grade	End of 8th grade	Upon Graduation
3.1	2-D Attributes: Describe, compare, and analyze attributes of two-dimensional shapes.	Properties: Define, classify and compare properties of solids and plane figures, including lines and angles.	Conjectures: Formulate and evaluate conjectures about geometric objects and their properties applying inductive reasoning with and without technology.
3.2	3-D Attributes: Describe attributes of three-dimensional shapes such as cubes, rectangular prisms, pyramids, cylinders, cones, and spheres.	Relationships: Determine congruence, similarity, and symmetry of objects using spatial reasoning in mathematics and in the contexts of art, science, and culture.	Applications: Use spatial reasoning and geometric models to solve contextual problems with and without technology in the contexts of art, science, and culture.
3.3	Transformations: Identify slides and flips of congruent figures using spatial reasoning within cultural and artistic contexts when relevant.	Transformations: Define, identify, and execute transformations including translations, rotations, reflections, and dilations which may be enhanced with technology.	Multiple Geometric Approaches: Identify, analyze, and use multiple geometric approaches including transformations in coordinate and synthetic settings from cultural and artistic contexts when relevant.

3.4	Measurement: Estimate and measure linear objects in metric units such as centimeters and meters and customary units such as half inch, inch, foot, and yard.	Measurement: Measure and compute angles, perimeter, area, surface area, and volume including the use of formulas and choosing appropriate metric or customary units.	Indirect Measurement: Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, Pythagorean Theorem, and angle relationships.
3.5	Area & Perimeter: Define and determine area and perimeter of common polygons using concrete methods such as grid paper, objects, or technology to justify their strategy.	Justification: Develop informal arguments to verify geometric relationships and solve problems in a variety of contexts with and without technology (such as the informal justification of the Pythagorean theorem)	Methods of proof: Establish the validity of geometric conjectures using deductive reasoning, indirect proof, and counter examples and critique arguments made by others.

Content Standard 4
Algebraic and Functional Reasoning: A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables. A student will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant **cultural contexts.**

Rationale

The study of algebra and functions opens doors and expands opportunities in numerous 21st century careers throughout many cultures. Students who generalize patterns and represent relationships in multiple ways develop significant understandings of mathematics and the use of quantitative reasoning in other disciplines. Algebra and functions are powerful tools for modeling real world relations and using those models to make informed decisions.

Benchmarks

	End of 4th grade:	End of 8th grade:	Upon Graduation
4.1	Patterns and Relations: Describe, extend, and make generalizations (including finding rules) about geometric or numeric patterns.	Patterns: Create and communicate using tables, graphs or diagrams, symbolic expressions, and verbal descriptions to represent, analyze, and generalize a variety of patterns involving numbers and operations.	Functions: Represent functions in a variety of ways including tables, graphs or diagrams, verbal descriptions, and symbolic expressions in recursive and explicit form. Select and justify an appropriate form for solving a given problem with and without technology.
4.2	Symbols: Use letters, boxes, or symbols to represent numbers in simple expressions or equations to demonstrate a basic understanding of variables.	Function: Identify linear and non-linear functional relationships and contrast their properties using tables, graphs, or equations with and without technology.	Symbolic Representation: Determine the appropriate symbolic representation of a given a contextual situation (e.g., variables and parameters in equations, inequalities, functions, and matrices).
4.3	Properties: Use number patterns to investigate	Solving: Use number properties and inverse	Solving: Solve a variety of equations, inequalities and systems of equations

	properties of numbers such as even or odd and operations such as multiplicative/additive id entities, commutative, associative, and distributive.	operations to solve single-variable two- step equations and inequalities.	and inequalities, justify the solution process, and interpret the solution in context.
4.4	Equivalence: Develop an understanding of equivalence by expressing numbers, measures, or numerical expressions involving operations in a variety of ways.	Equivalence: Recognize, simplify, and generate equivalent forms for algebraic expressions justifying their steps with properties of operations.	Transforming Functions: Analyze the effects of transformations on families of functions and recognize their characteristics. Represent functions in equivalent forms. Use equivalent forms of functions to identify and perform transformations.
4.5	Modeling: Model problem situations with manipulatives or technology and use multiple representations such as words, pictures, tables, or graphs to draw conclusions in cultural contexts when relevant.	Modeling: Identify and compute rate of change/slope and intercepts from equations, graphs, and tables; model and solve contextual problems involving linear proportions or direct variation in cultural contexts when relevant.	Modeling: Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology in cultural contexts when relevant.